Distributed computing of coverage in sensor networks by homological methods

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- Each sensor is equipped with a processor and radio transmitter. The radio transmitter gives the sensor the ability to communicate with nearby sensors.

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  4. $r_c \geq \frac{r_b}{\sqrt{3}}$
- One of the fundamental problems concerning sensor networks is determining whether every point in a given bounded set $\mathcal{D} \subset \mathbb{R}^2$ is covered by the network, i.e. if

$$\mathcal{D} \subset \bigcup_{s \in S} B(s, r_c).$$

- The importance of the coverage problem in the sensor networks: security, mine field sweeping, ad hoc communication networks.

Assumptions.

- A *sensor network graph* associated with a sensor network (S, r_b, r_c) is a graph whose vertices coincide with S and whose edges are doubletons $\{s_1, s_2\} \in \binom{S}{2}$ such that $d(s_1, s_2) \leq r_b$. We denote such a graph by $G_{(S, r_b, r_c)}$.

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- By a *fence* of a sensor network (S, r_b, r_c) we mean a subset $F \subset S$ such that the geometric realization of the subgraph of $G_{(S,r_b,r_c)}$ induced by F is a Jordan curve in $\mathbb{R}^2$.

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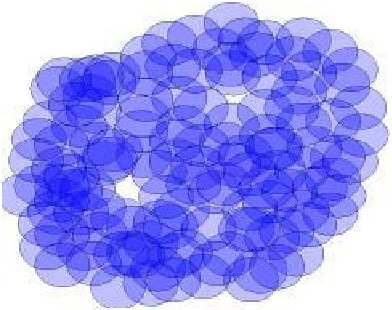
- The presented algorithm verifies the partial criterion of the coverage for subsets $\mathcal{D} \subset \mathbb{R}^2$ which constitute the bounded component of the complement of the Jordan curve of a fence.

Example.

Figure: An example of a region covered by a sensor system. [2]



The homological criterion

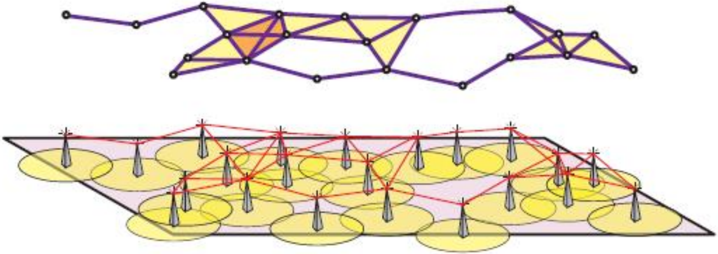
The coverage criterion based on the relative homology of the Rips complexes.

Theorem (Coverage criterion, see [2])

Let (S, r_b, r_c) *be a sensor network with a fence* F *and let* $\mathcal{R} := \mathcal{R}_{r_b}(S)$ *and* $\mathcal{F} := \mathcal{R}_{r_b}(F)$ *be the associated Rips complexes. If there exists a homology class* $[\alpha] \in H_2(\mathcal{R}, \mathcal{F})$ *such that* $\partial\alpha \neq 0$*, then* $\mathcal{D}$*, the bounded component of the Jordan curve of the fence, satisfies*

$$\mathcal{D} \subseteq \bigcup \{B \subset \mathbb{R}^2 \mid B = B(s, r_c), s \in S\}.$$

Example of a Rips complex [4]



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- ...which is not easy and not effective for large complexes.

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- Doing algebraic homology computations in the distributed way is not easy.
- Why not use the reduction procedures, that can be implemented in a purely combinatorial way?
- In some cases the (co)reductions can reduce the complex to homology generators.

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- All decisions about a simplex σ are taken by the lower sensor controlling σ.

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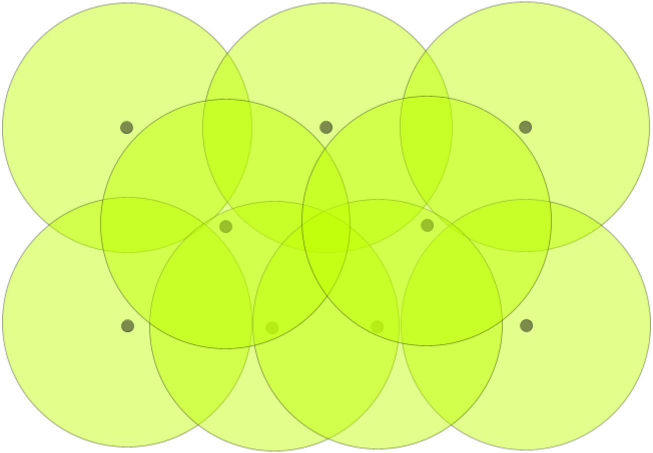
- detect all their neighbors and the labels of the neighbors of their neighbors,
- this information is enough to build the Rips complex associated with the sensor network, such that the information about simplex σ is kept in the two sensors,
- no further communication is needed to construct the Rips complex.

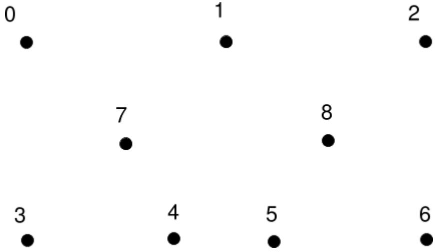
Example.

Figure: An example of a sensor system ($r_c = 2.5$, $r_b = 4$).



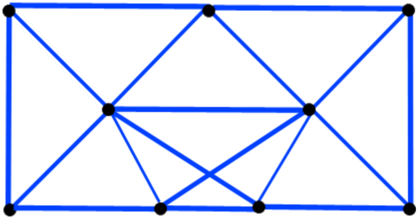
Example.

Figure: 0-simplices.



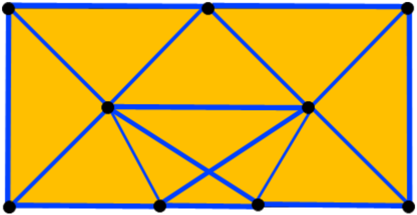
Example.

Figure: 1-simplices.



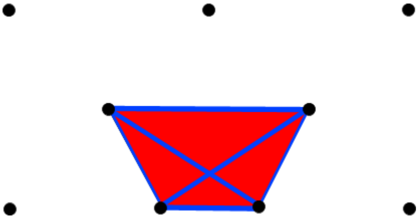
Example.

Figure: 2-simplices.



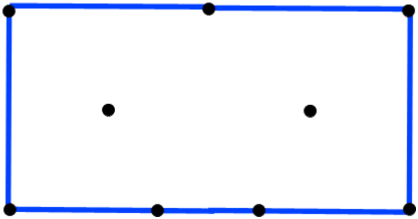
Example.

Figure: 3-simplices.



Example.

Figure: Fence subcomplex.



Example.

Figure: All simplices.

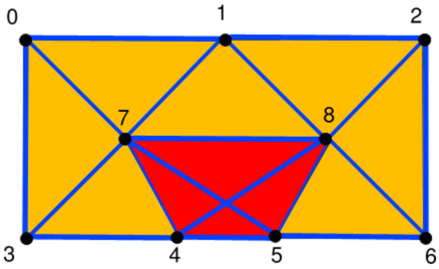


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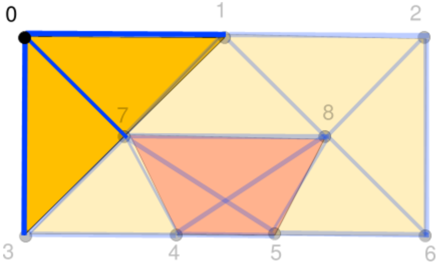
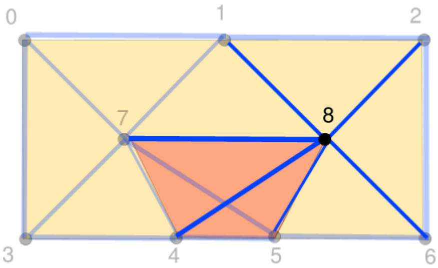
Figure: Simplices stored in sensor 0.



Example.

Figure: Simplices stored in sensor 8.



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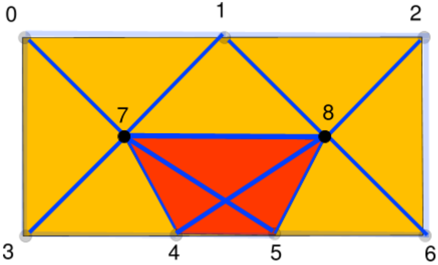
- So we simply remove the 0 and 1 dimensional simplices that belong to the fence from the constructed Rips complex.
- This can be done by the sensors themselves.

Example.

Figure: The complex without the fence.



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- To do so the combination of the reduction and coreduction algorithm will be used.

(Co)reduction algorithms

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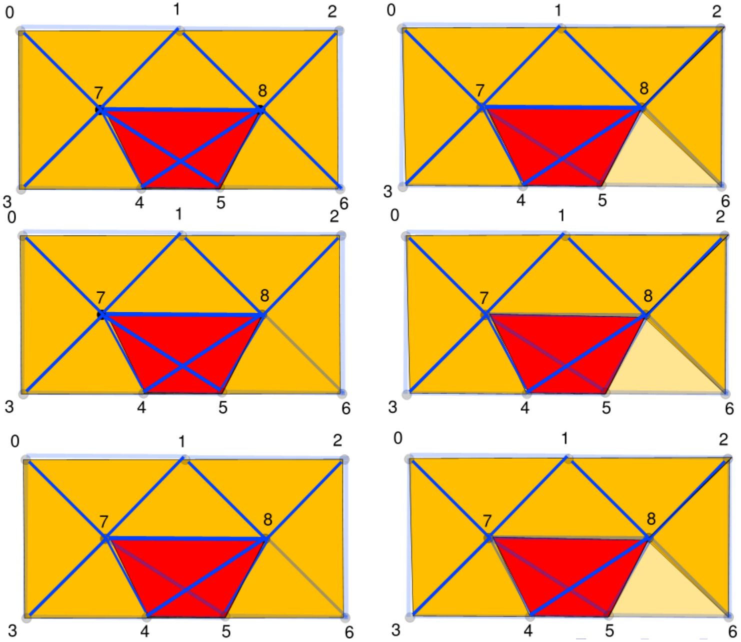
- We perform the reduction process in such a way that simplex σ may be reduced only by the lower sensor controlling it.

Example.



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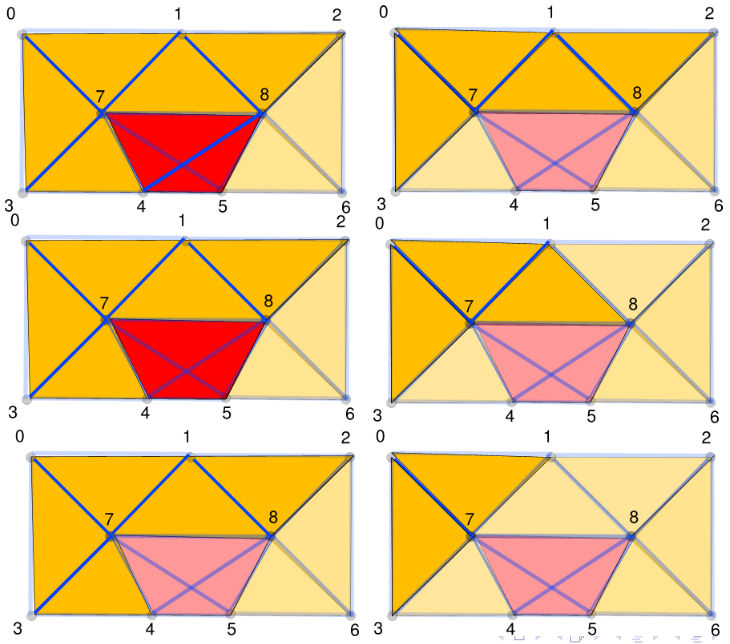
Example.



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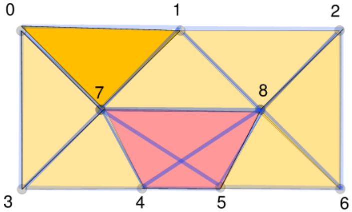

Implementation

The algorithms presented in the talk have been implemented in a Sensor Network Simulator. The Sensor Network Simulator executes the presented algorithms on each sensor in a separate thread, so that the simulations may be performed with only a few or even one processor unit.

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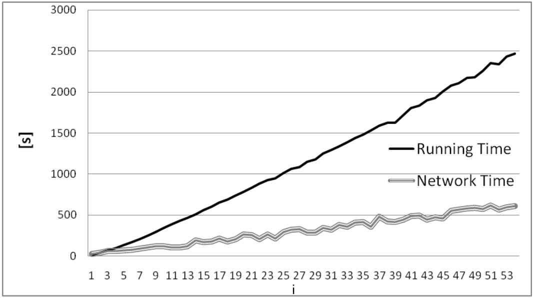
- We define the network efficiency as the ratio

$$\texttt{Network Efficiency} := (\texttt{Network Time})/(\texttt{Running Time})\cdot 100\%.$$

Linear test

1. L_0 is a base test case consist of 23 sensors randomly placed on a 4×4 units rectangle with the communication radius fixed to 2 units with a rectangular fence around the network,
2. L_i is created from L_{i-1} by placing a new copy of the base test on the right side of L_{i-1} with a new rectangular fence around the network.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Running Time
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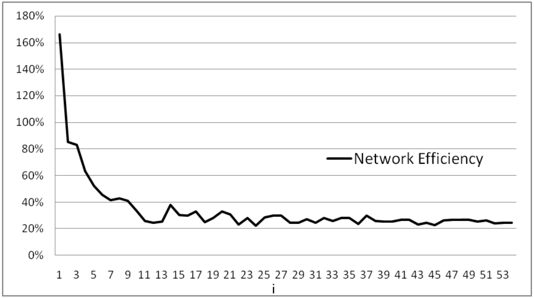
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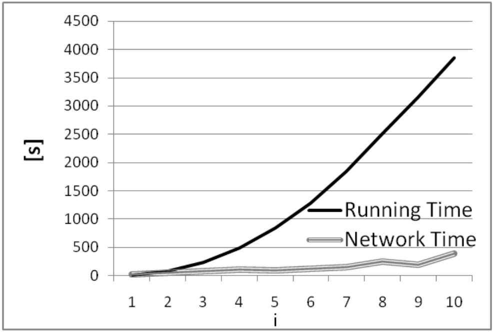
Square test

1. S_0 is the base test case with a rectangular fence around the network,
2. S_i is created from S_{i-1} by placing three new copies of the base test: on the right, on the bottom side, and on the bottom-right side of S_{i-1}. A new rectangular fence around the network is created.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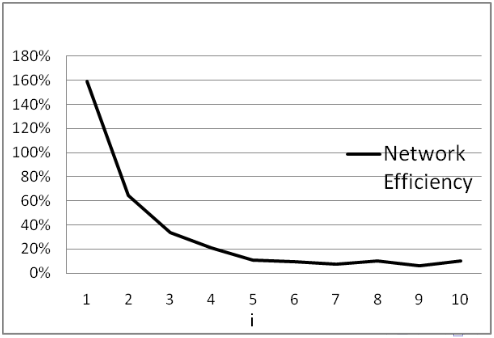
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Can this work? Can we prove that the algorithm is correct?

- In the papers dealing with reduction and coreductions proofs of the correctness of the sequential algorithms provided.
- distributed version of these algorithms used here.
- How to prove the correctness of the distributed version of the algorithm?
- This is what I am going to talk about.

S-complexes, algorithmic reformulation of the chain complexes.

- $\mathcal{R}$ - finite set with gradation,
- $\kappa : \mathcal{R} \times \mathcal{R} \to R$ – incidence map,
- $(\mathcal{R}, \kappa)$ is a S-complex iff $(R(\mathcal{R}), \partial\kappa)$ with $\partial\kappa(\sigma) = \sum_{\tau \in \mathcal{R}} \kappa(\sigma, \tau)\tau$ is a free chain complex with a base $\mathcal{R}$,
- operation done on a basis $\mathcal{R}$, subroutine κ for indices provided,
- a subset $\mathcal{R}' \subset \mathcal{R}$ referred to as regular, if $(\mathcal{R}', \kappa|_{\mathcal{R}' \times \mathcal{R}'})$ is a subcomplex of $(\mathcal{R}, \kappa)$ (well, almost, let's keep it simple to get the idea...),
- after removing a reduction pair from the complex a regular subset is obtained.

Chain maps related to each reduction.

- (τ, σ) – a reduction pair in complex $\mathcal{R}$.
- Map $\eta_{(\tau,\sigma)} : C(\mathcal{R} \setminus \{\tau, \sigma\}) \to C(\mathcal{R})$ inducing isomorphism in homology is given.
- For $c \in C(\mathcal{R} \setminus \{\tau, \sigma\})$, $\eta_{(\tau,\sigma)}(c) = c - \frac{\langle \partial c, \tau \rangle}{\langle \partial \sigma, \tau \rangle} \sigma$.
- Building blocks for the map used in the further formal analysis.

State graph of the S-complex – the formal model.

- $G_{\mathcal{R}} = (V, E)$,
- V consist of regular subsets of $\mathcal{R}$,
- for $\mathcal{K}, \mathcal{K}' \in V$ $(\mathcal{K}, \mathcal{K}') \in E$ iff $\mathcal{K}' = \mathcal{K} \setminus \{\tau, \sigma\}$ for (τ, σ) being a reduction pair in $\mathcal{K}$,
- *Reduction path* ψ - directed path in $G_{\mathcal{R}}$ joining $\mathcal{K}$ and $\mathcal{K}'$,
- need some more specific type of reduction paths to show, that reductions can be applied in a distributed way.

Simple reduction paths.

- Let $\mathcal{A}$ be the set of all reduction pairs in the initial complex $\mathcal{R}$,
- before a reduction of a pair $(\sigma, \tau) \notin \mathcal{A}$ can be made, some of the reductions in $\mathcal{A}$ have to be already made,
- this leads to the idea of simple reduction paths:
- *Reduction path* $(\tau_1, \sigma_1), \ldots, (\tau_n, \sigma_n)$ joining $\mathcal{K}$ and $\mathcal{K}'$ – *simple* if (τ_i, σ_i) is a reduction pair in $\mathcal{K}$ for every $i \in \{1 \ldots n\}$.
- ϕ and ψ s.r.p. sharing the initial vertex - equivalent iff. reductions along ϕ are permutation of reductions along ψ.

Equivalent simple reduction paths induce the same maps in homology.

- $\psi = (\tau_1, \sigma_1) \dots (\tau_n, \sigma_n)$ - simple reduction path between $\mathcal{K}$ and $\mathcal{K}'$,
- induces map in homology $H(\psi) = \eta_{(\tau_1,\sigma_1)} \circ \dots \circ \eta_{(\tau_n,\sigma_n)}$,

- Theorem
  ψ and ϕ – equivalent simple reduction paths joining $\mathcal{K}$ and $\mathcal{K}'$ in the state graph, then $H(\psi) = H(\phi)$,
  - therefore the map in homology does not depend on the choice of a simple reduction path between $\mathcal{K}$ and $\mathcal{K}'$,
  - reductions on the simple reduction paths have disjoint supports,
  - they can be reduced in a distributed way.

Equivalent simple reduction paths induce identical maps in homology.

- Idea of a proof:
- ψ and ϕ differs by a permutation of reduction pairs.
- transpose one of them to get the other one.
- Only the reduction and coreduction pairs considered,
- sufficient to prove for any simple reduction path of length 2 (simple induction).

Distributed reduction model, idea.

- $\mathcal{R}$ – initial, $\mathcal{R}'$ – final complex in the course of reductions,
- usually the reduction process is long and deep,
- consequently no simple reduction path from $\mathcal{R}$ to $\mathcal{R}'$,
- **Idea:** divide, for the sake of proof, the whole execution of the algorithm to some levels,
- $\mathcal{R} \xrightarrow{p_1} \mathcal{K}_1 \xrightarrow{p_2} \cdots \xrightarrow{p_n} \mathcal{R}'$,
- each arrow p_i represents a simple reduction path. In homology induce isomorphism,
- explicit formula for the map p_i and its inverse are given.

Distributed reduction model, aim.

- Suppose, that each sensor stores information for each reduction pair if it was reduced as a reduction or coreduction pair,
- $\mathcal{A}$:=set of all reductions made by all the sensors in the course of running the algorithm,
- aim: show that $H(\mathcal{R} \setminus \mathcal{A}) \backsimeq H(\mathcal{R})$.

Correctness of the reductions via parallel algorithm.

- For $(\tau, \sigma) \in \mathcal{A}$ reduced as coreduction pair

$$\alpha(\tau, \sigma) := \{(\overline{\tau}, \overline{\sigma}) \in \mathcal{A} | \{\overline{\tau}, \overline{\sigma}\} \cap (\mathsf{bd}_{\mathcal{R}}\, \sigma \setminus \tau) \neq \emptyset\},$$

- for $(\tau, \sigma) \in \mathcal{A}$ reduced as reduction pair

$$\alpha(\tau, \sigma) := \{(\overline{\tau}, \overline{\sigma}) \in \mathcal{A} | \{\overline{\tau}, \overline{\sigma}\} \cap (\mathsf{cbd}_{\mathcal{R}}\, \tau \setminus \sigma) \neq \emptyset\},$$

- in "α-sets" reductions that need to be applied prior to (τ, σ) are stored,
- define $\lambda(\tau, \sigma) := 1 + max\{\lambda(\overline{\tau}, \overline{\sigma}) | (\overline{\tau}, \overline{\sigma}) \in \alpha(\tau, \sigma)\}$,
- the function λ is a tool to define the following simple reduction paths.

Correctness of the reductions via parallel algorithm.

- $\mathcal{A}_n := \{(\tau, \sigma) \in \mathcal{A} | \lambda(\tau, \sigma) = n\}$,
- $\mathcal{K}^0 := \mathcal{R}$,
- $\mathcal{K}^n := \mathcal{K}^{n-1} \setminus |\mathcal{A}_n|$,
- **Theorem:** $\mathcal{K}^n$ is an S-complex.
- **Theorem:** if $(\sigma, \tau) \in \mathcal{A}_n$, then (σ, τ) is an reduction or coreduction pair in $\mathcal{K}^{n-1}$,
- global isomorphism between $H(\mathcal{K})$ and $H(\mathcal{K} \setminus \mathcal{A})$ can now be defined as a composition of isomorphisms between $\mathcal{K}^i$ and $\mathcal{K}^{i+1}$ (simple reduction paths!)

The conjecture.

Conjecture

For the relative homology of a Rips complex, built from a planar point set, the sequence of reductions and coreductions reduce the complex up to its homology generators.

- The idea behind this – even if the dimensions of simplices in the Rips complex are high, the topology of it almost "planar" (in the planar case the reductions can do all the job) ...
- this is what we see from numerical experiments,
- even if the Conjecture is not true, the complex that remains after (co)reductions is very small...
- and can be gathered by one of the sensors and the homology can be computed.

Verifying the covering.

- Ghrist–de Silva criterion requires information about the representatives of homology generators in the complex $\mathcal{R}$ (the initial one),
- According to the conjecture we have the homology generator $c \in C_2(\mathcal{R} \setminus \mathcal{A})$,
- information about the applied coreductions allows us to find the isomorphic companion of c in $C_2(\mathcal{R})$,
- explicit formulas for the isomorphisms on the chain level are known,
- they can be implemented as a distributed algorithm,
- this algorithm restores the representant of the generator from $C_2(\mathcal{R} \setminus \mathcal{A})$ to $C_2(\mathcal{R})$.

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The end.

Thank you for your attention!